Phase diagram of a spinor exciton-polariton condensate in a disordered microcavity in the presence of a magnetic field

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We establish a phase diagram of a spinor exciton-polariton condensate in a disordered microcavity in the presence of an external magnetic field. We find that the combination of the full paramagnetic screening and Anderson localization leads to the formation of a condensed phase having both localized and superfluid components. This is reflected by different dispersions of elementary excitations for the two polarization components.

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I. INTRODUCTION

Exciton polaritons (polaritons) are the new eigenstates formed from quantum-well excitons strongly coupled with cavity photons. Like excitons, in the low-density limit they behave as two-dimensional (2D) weakly interacting bosons,[1] having extremely small effective mass inherited from cavity photons. Stimulated scattering toward the ground state has been demonstrated[2,3] and their Bose Einstein condensation (BEC) reported.[4–7] Because of the finite radiative life time of polaritons, the relaxation kinetics can play an important role in the formation of a polariton condensate. It has been shown[8] that increasing the temperature and going toward positive exciton-photon detuning can strongly improve the relaxation rate of polaritons, which becomes much faster than their life time. In that regime a quasiequilibrium distribution of polaritons forms and the phase transition toward a Bose condensed phase can be described by the standard thermodynamic theory. In the quasiequilibrium regime cavity polaritons are expected to undergo Kosterlitz-Thouless phase transition toward superfluidity,[9] which was not experimentally observed in CdTe (Ref. 5) and GaN (Ref. 6) microcavities, strongly affected by structural disorder and localization. Using the Gross-Pitaevskii equation and taking into account a realistic disorder potential we have recently shown[10] that with increase in the density, polaritons first undergo a phase transition toward an Anderson glass phase,[11] characterized by a long spatial coherence, a flat dispersion, and delocalizing role of the interactions. The phase-coherent condensate is localized in minima of the disorder potential and demonstrates no superfluid behavior. A further increase in the density leads to a blueshift of the chemical potential μ, the repulsive polariton-polariton interactions resulting in percolation process and phase transition toward a superfluid state. This interpretation has been recently supported, first by the confirmation of the strong role of the disorder on the nature of the polariton condensate[12] and second by the observation of a Bogoliubov-type dispersion of the elementary excitations in a GaAs cavity,[13] much less affected by disorder than GaN or CdTe structures. The Anderson glass has also been observed recently for atomic condensates,[14] which has warmed up the general interest in the subject.

Another important topic in a domain of quantum microcavities is spin dynamics of polaritons which has been at the heart of an intense theoretical and experimental activities in the past years.[1] Being composed of heavy-hole exciton and cavity photon, polaritons are spinor quasiparticles with two possible spin projections ± 1 on the structure growth axis. In the domain of polariton BEC, the polarization of the system plays the role of the order parameter of the phase transition.[15] Its orientation is controlled by an effective in-plane magnetic field arising from possible cavity anisotropies,[16] real applied magnetic field,[17] and spin-dependent polariton-polariton interactions,[18] which are usually strongly spin anisotropic. The interaction constant α_1 between particles having the same spin projections on structure growth axis (triplet configuration) is much larger than α_2, the interaction constant between particles having opposite spin projections (singlet configuration). As a result, in the absence of the external magnetic field, it is energetically preferential for the condensate to be linearly polarized, as observed experimentally by several groups.[7,16,19] In bulk GaN cavities, the situation is different because of the strong mixing in the polariton state of the A and B excitons based on heavy and light holes which makes the polariton-polariton interaction spin independent. As a result, there is no preferential orientation for the polarization of the condensate, which can take any direction from one experiment to another.[6]

One of the intriguing polarization-related phenomena recently predicted for spin-anisotropic polariton-polariton interaction is the full paramagnetic screening, also known as spin-Meissner effect.[20] A magnetic field applied along the structure growth axis would lead to a Zeeman splitting of circularly polarized states. The noninteracting condensate is expected therefore to occupy the lowest available energy state, redshifting its chemical potential versus magnetic field and staying fully circularly polarized. The situation is however different if spin anistropy of polariton-polariton interactions is accounted for. Indeed, the transition from a linear to a circular polarization would cost a macroscopic energy $(\alpha_1-\alpha_2)n^2/2$ which, at low fields and high enough polariton concentrations, is larger than the magnetic energy $\Omega=\mu_B nH$ that the condensate would gain if it were circularly polarized. As a result, below a critical field, the Zeeman splitting is fully suppressed. The chemical potential remains independent of the magnetic field and the condensate is elliptically polarized, with a circular polarization degree progressively increasing versus magnetic field. Above the critical field the magnetic energy becomes larger than the interaction energy and the Zeeman splitting is recovered. The condensate becomes circularly polarized and redshifts with increase of the magnetic field.

So far, the interplay between spin response of a condensate and its Anderson localization due to the disorder has never been analyzed, neither for the case of polaritons nor for cold atoms. In this paper we establish the phase diagram of spinor polariton condensates in the presence of structural disorder and external magnetic field. We find a peculiar condensed phase, where the condensate, elliptically polarized as a whole, is strongly inhomogeneous in space with one circular component localized in the minima of the disorder potential and another one delocalized and superfluid. It should be noted that similar effects can be observed in a domain of BEC of cold atoms possessing nonzero spin[21] in disordered optical lattices. The difference will be that the number of possible spin projections for an atom exceeds two (e.g., $-1,0,+1$ for spin-1 atoms). Thus, the variety of phases will be increased as compared to the case of polaritons considered here.

II. SINGLE TRAP

Let us first consider the case $T=0$ K, assuming for simplicity that particles have an infinite mass (Thomas-Fermi approximation). We consider a single rectangular potential well of a depth V_0, surface S_0 in system of total surface S, placed in a magnetic field B applied parallel to the structure growth axis. The interaction of polaritons in the triplet configuration is described by the constant which can be estimated[22] as $\alpha_1 \approx 3E_B a_B^2$, and the singlet interaction constant α_2 which is much smaller[23] than α_1 is neglected. At zero magnetic field and low particle density, the polariton condensate is linearly polarized with the same number of particles for both circular components ($N_{\sigma+}=N_{\sigma-}=N/2$) and localized in the region of a potential trap. Increasing the particle density, the condensate delocalizes and becomes superfluid when the chemical potential reaches the edges of the trap $\mu=-V_0+\alpha_1 N/2S_0=0$ (we take the potential energy outside the trap as the zero reference), which occurs when the total number of polaritons becomes $N=2V_0S_0/\alpha_1$.

We now consider the case of a finite magnetic field sketched in Fig. 1. Since the polaritons are electrically neutral, the magnetic field does not change the confining potential and thus does not directly affect the localization. If the number of polaritons in the system is small, the magnetic field exceeds the critical spin-Meissner field $\Omega > \Omega_c = \alpha_1 N/S_0$. The circular polarized states become split and the localized condensate occupies the lowest of these two states becoming circularly polarized ($N=N_{\sigma^+}$). The increase in N increases the chemical potential of the σ^+-polarized condensate, as

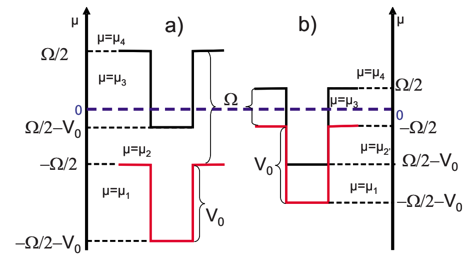

FIG. 1. (Color online). Potential profile seen by σ^+ (red/dark gray) and σ^- polaritons (black) in the presence of a magnetic field yielding a Zeeman splitting Ω. The dashed lines shows the position of the chemical potential for different phases. (a) $V_0<\Omega$, $\mu=\mu_1$: σ^+ localized condensate; $\mu=\mu_2$ σ^+ superfluid condensate; $\mu=\mu_3$: new condensed phase with the σ^- component localized and the σ^+ delocalized; $\mu=\mu_4$: elliptically polarized superfluid phase. (b) $V_0>\Omega$, $\mu=\mu_1$: σ^+ localized condensate; $\mu=\mu_{2'}$: localized elliptically polarized condensate; $\mu=\mu_3$ new condensed phase with the σ^- component localized and the σ^+ delocalized; $\mu=\mu_4$: superfluid phase elliptically polarized.

$$\mu = \mu_1 = -V_0 - \Omega/2 + \alpha_1 N/S_0 \tag{1}$$

provided that $\mu < -\Omega/2$. The energy of the unpopulated σ^- state remains unchanged.

Depending on the ratio between the Zeeman splitting and the confining energy, two different situations can take place. If $V_0<\Omega$ [Fig. 1(a)], the increase in the density leads to a delocalization of σ^+ condensate ($\mu=\mu_1$) when the condition $\mu \geq -\Omega/2$ is fulfilled. At this point the condensate becomes a circularly polarized superfluid with a chemical potential,

$$\mu = \mu_2 = -\Omega/2 + \alpha_1\left(N - \frac{V_0 S_0}{\alpha_1}\right)/S \tag{2}$$

which is valid in the range $\Omega/2 - V_0 \geq \mu \geq -\Omega/2$. Here S is the total size of the system. With a further increase in N, the local density in the trap region becomes large enough so that the spin-Meissner effect takes place there and σ^- component appears when $\mu \geq \Omega/2 - V_0$. As a whole, the condensate becomes elliptically polarized with some finite percentage of the σ^- component and the chemical potential,

$$\mu = \mu_3 = -\Omega/2 + \alpha_1\left(N_{\sigma^+} - \frac{V_0 S_0}{\alpha_1}\right)/S = \Omega/2 - V_0 + \frac{\alpha_1 N_{\sigma^-}}{S_0} \tag{3}$$

which is valid while $\Omega/2 \geq \mu \geq \Omega/2 - V_0$. However, the σ^- component remains localized in a trap region and does not participate in the superfluidity. The condensate has therefore one localized component and one superfluid. The polarization in this case is elliptic inside the trap and circular outside. A further increase in the density allows a delocalization of the σ^- component as well which occurs when $\mu \geq \Omega/2$. Above this value the chemical potential reads

$$\begin{aligned}\mu &= \mu_4\\ &= -\Omega/2 + \alpha_1\left(N_{\sigma^+} - \frac{V_0 S_0}{\alpha_1}\right)/S\\ &= \Omega/2 + \alpha_1\left(N_{\sigma^-} - \frac{V_0 S_0}{\alpha_1}\right)/S \end{aligned} \tag{4}$$

under the condition $\mu \geq \Omega/2$.

The second situation corresponds to $V_0 > \Omega$ [Fig. 1(b)]. For small occupancies corresponding to $\mu = \mu_1$ the condensate is fully σ^+ polarized and localized, which with the density increase transforms into a localized elliptically polarized condensate with a spin-Meissner effect taking place and the chemical potential,

$$\mu = \mu_{2'} = -V_0 - \Omega/2 + \alpha_1 N_{\sigma^+}/S_0 = -V_0 + \Omega/2 + \alpha_1 N_{\sigma^-}/S_0 \tag{5}$$

valid in the range $-\Omega/2 \geq \mu \geq \Omega/2 - V_0$. Further increase in the density leads to the transition at $\mu \geq -\Omega/2$ into a phase characterized by delocalized and superfluid σ^+ component and localized σ^-, corresponding to $\mu = \mu_3$. Finally, at very high densities, when $\mu \geq \Omega/2$ both components become delocalized ($\mu = \mu_4$).

III. RANDOM DISORDER POTENTIAL

The generalization of this picture to the case of a two-dimensional random disorder potential is straightforward. In this case the blueshift for each polarization component is still given by the expression $\alpha_1 N_{\sigma^\pm}/S_{0,\sigma^\pm}$ but now the surface $S_{0,\sigma^\pm}$ occupied by each circular component of the condensate increases with the number of particles in this component since more and more minima of the potential become filled. For a Gaussian distribution of the disorder potential with a zero mean value and root-mean-square fluctuation V_0 the surface $S_{0,\sigma^\pm}$ occupied by the states having an energy smaller than a given value E is given by $S_0(E) = \frac{S}{2}[1 + \mathrm{erf}(E/V_0)]$. For a given chemical potential the number of particles in each component is therefore given by

$$\begin{aligned} N_{\sigma^\mp} &= \int_{-\infty}^{\mu \pm \Omega/2} \frac{S}{2\alpha_1}[1 + \mathrm{erf}(E/V_0)]dE\\ &= \frac{S}{2\alpha_1}\Bigg((\mu \pm \Omega/2)\{1 + \mathrm{erf}[(\mu \pm \Omega/2)/V_0]\}\\ &\quad + \frac{V_0}{\sqrt{\pi}} e^{-(\mu \pm \Omega/2)^2/V_0^2}\Bigg). \end{aligned} \tag{6}$$

Dividing the result by S one obtains the particle density of each component as a function of the chemical potential μ. The corresponding dependence is superlinear because the surface occupied by the condensate is increasing with μ, and therefore more particles are necessary in order to have the same increase in the energy. According to the percolation theory, the delocalization threshold in 2D for a potential with a symmetric distribution is reached when mean value of the repulsion energy becomes equal to V_0. Similarly to the case of a single trap previously discussed, the delocalization of the σ^+ component occurs if $\mu \geq -\Omega/2$. The appearance of the σ^- component occurs when $\mu \geq \Omega/2 - V_0$ and it becomes delocalized when $\mu \geq \Omega/2$. As in the case of a single potential trap, we have two possible situations of $V_0 < \Omega$, and $V_0 > \Omega$. In both cases, there is a regime when one component is delocalized and the other is localized and polarization varies from one point to another in the real space. After chemical potential passes the threshold $\mu = \Omega/2$, both components become delocalized and polarization remains elliptic tending to linear in the infinite density limit, when both the disorder and the magnetic field can be neglected compared to the interaction energy.

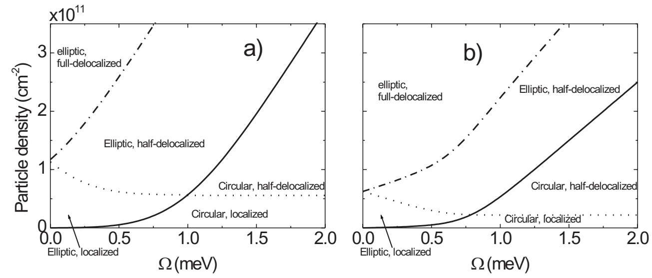

FIG. 2. Phase diagram for a polariton condensate in a disordered system under magnetic field corresponding to a bare exciton Zeeman splitting Ω: (a) infinite mass approximation (analytic solution) and (b) finite mass (numeric solution of Gross-Pitaevskii equations). Different phases are indicated by text labels. Phase boundaries are indicated with different line styles described in the text.

Now we can plot the phase diagram of the disordered polariton system under magnetic field [Fig. 2(a)] which maps all the phases previously discussed. The solid line is given by the condition $\mu = \Omega/2 - V_0$, below this line there is only one polarization component present. The dotted line is given by the condition $\mu = -\Omega/2$ corresponding to a percolation transition. The dash-dotted line is given by the condition of delocalization of the minority component $\mu = \Omega/2$. All the calculations have been performed for a typical CdTe microcavity parameters, similar to those used in previous calculations.[10] The maximal Zeeman splitting of 2 meV shown in this diagram could be achieved for a magnetic field of about 20 T.

After studying the simplified system analytically, we have carried out the numerical analysis corresponding to the case of finite effective mass m which takes into account the kinetic energy. We minimize the free energy of the system of interacting polaritons in a disorder potential:

$$\begin{aligned} F = \int \mathbf{dr}\Bigg[&\frac{\hbar^2}{2m}(\psi_+^*\nabla^2\psi_+ + \psi_-^*\nabla^2\psi_-) + V(|\psi_+|^2 + |\psi_-|^2)\\ &+ \frac{\alpha_1}{2}(|\psi_+|^4 + |\psi_-|^4) + \frac{\Omega}{2}(-|\psi_+|^2 + |\psi_-|^2)\Bigg]. \end{aligned} \tag{7}$$

The results are shown in Fig. 2(b). Qualitative behavior is the same as in the infinite mass approximation but all thresholds are reduced due to the kinetic-energy term. One can clearly observe that the effect of the finite mass is stronger for the localization-delocalization transition than for the circular-elliptic transition because the particles with a light

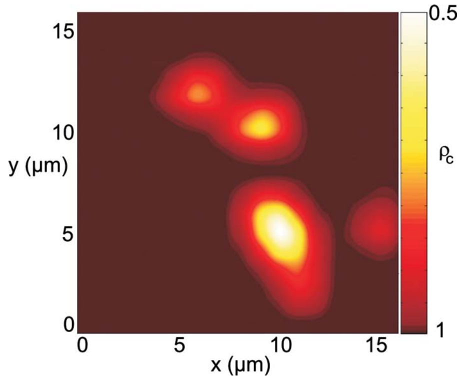

FIG. 3. (Color online). Calculated spatial distribution of circular polarization degree ρ_c for the semilocalized phase.

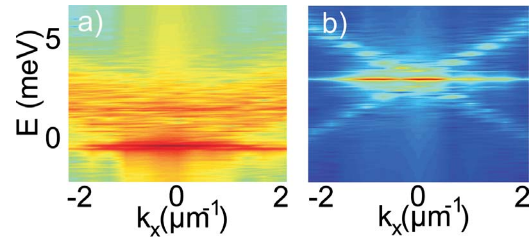

FIG. 4. (Color online). Dispersions of the excitations of the two polarization components of a semilocalized condensate (a) localized component and (b) delocalized component.

effective mass effectively penetrate into the barriers whereas the total kinetic energy may be relatively small as compared to the magnetic energy.

The peculiar phase with a semidelocalized condensate and polarization varying from point to point deserves special attention. An example of a spatial distribution of circular polarization degree ρ_c for a particular realization of a disorder potential is given in Fig. 3. The regions with the localized σ_- component show a polarization degree different from 1 whereas outside these regions the condensate is fully circularly polarized. A sufficient increase in the particle density would lead to the percolation of the σ_- component; the regions with elliptic polarization would become interconnected.

IV. SUPERFLUID BEHAVIOR

In order to check the superfluid properties of both polarization components in this case, we have studied the dispersions of the excitations of these two polarization components. This has been done by adding a small perturbation to the wave function of the ground state previously found by minimizing the free energy. Then we were solving the time-dependent Gross-Pitaevskii equations for polaritons taking the perturbed wave function as an initial condition,

$$i\hbar\frac{\partial}{\partial t}\psi_+(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) - \frac{\Omega}{2} + \alpha_1|\psi_+(\mathbf{r},t)|^2\right]\psi_+(\mathbf{r},t),$$

$$i\hbar\frac{\partial}{\partial t}\psi_-(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + \frac{\Omega}{2} + \alpha_1|\psi_-(\mathbf{r},t)|^2\right]\psi_-(\mathbf{r},t). \tag{8}$$

The resulting time-dependent wave functions of the two polarization components were Fourier transformed in time and space domains in order to get $|\psi(\mathbf{k},\omega)|^2$ plotted in Fig. 4. This figure shows clear signatures of both regimes: (a) localized for one component with parabolic dispersion containing a flat part (as seen in experiments[5]) and (b) delocalized superfluid with a characteristic linear dispersion (also recently observed experimentally[13]). This is somewhat analogous to He^4 below a λ point, where normal and superfluid components coexist. The peculiarity of our case is that for polaritons these are the two polarization components of a single condensate which show different behavior.

A realistic nonmagnetic CdTe cavity at magnetic fields up to 10 T with a relatively strong disorder with $V_0 \sim 0.5-1$ meV would be possibly described by the left part of the phase diagram (small Ω).

With the increasing particle density (pumping intensity), the following phases would be observed for such cavity: (1) circularly polarized localized phase (Anderson glass), showing flat dispersion and no superfluidity; (2) elliptically polarized localized phase (Anderson glass), showing flat dispersion for both components, no superfluidity, and a polarization varying from point to point; (3) elliptically polarized semilocalized phase, showing flat dispersion and no superfluidity for one component and linear dispersion with superfluid behavior for the other, and a polarization varying from point to point. The density range where the semilocalized phase could be observed is proportional to the value of the Zeeman splitting of a bare exciton. For practical reasons, semimagnetic cavities showing strong coupling[24] might be interesting for the observation of this phase; (4) elliptically polarized superfluid phase showing linear dispersion and superfluid behavior for both components with polarization tending to linear in the infinite density limit.

For systems with weak disorder or at stronger magnetic fields one would observe a different phase (2): (2′) circularly polarized superfluid phase, showing linear dispersion and superfluidity with constant polarization degree (fully circularly polarized). The other circular component is completely absent.

Therefore, in this case the localized/superfluid phases will be alternating. This manifests itself in the nonmonotonous behavior of the superfluid fraction, showing a small minimum for the phase (3), associated with the appearing localized component. This minimum should not be observed for weaker magnetic fields, where the phase (2) is not superfluid.

V. CONCLUSIONS

To conclude, we have studied a polariton condensate in a disordered microcavity in the presence of an external magnetic field both analytically (infinite mass case) and numerically (finite mass case). A peculiar phase separating the Anderson glass phase and the superfluid phase is found. It is characterized by different properties of the two polarization components of the condensate, one of them being localized and the other delocalized. We have shown that the dispersion of the localized phase is parabolic whereas the dispersion of the delocalized component is linear.

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